GPR ANTEENA POSITION AND ORIENTATION ESTIMATION USING STRAPDOWN INERTIAL NAVIGATION

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ABSTRACT

This paper discusses the possible use of strapdown inertial navigation for real-time ground penetrating radar (GPR) antenna positioning and orientation estimation along arbitrary three-dimensional acquisition lines. Strapdown inertial navigation theory has been studied extensively in the literature for aircraft, missile and space navigation. Here, we give an overview of the theory as it applies to the antenna position and orientation problem. This includes the definition of the relevant coordinate frames and attitude parameters, a discussion of the measured acceleration and angular velocity, and a description of the four primary computational tasks pertinent to strapdown inertial navigation. These are the initial alignment of the system, the integration of angular velocity into attitude (attitude updating), the acceleration transformation and integration into velocity (velocity updating) and the integration of velocity into position (position updating). The key elements of using a low-grade versus a high-grade inertial measurement unit (IMU) are pointed out. The actual performance of a commercially available low-grade IMU is evaluated based on a series of navigation experiments. The experimental results show that the tested IMU is far from being accurate enough for completely self-contained antenna positioning and that the precise calibration for scale factors, biases and axis misalignments is vital. The observed orientation accuracy (error of less than 1 degree after 60 seconds) suggests the integration of the tested IMU with odometry, extending the applicability of the latter to environments with topography or where changing of the profile direction due to obstacles is necessary. Another possible use of low-grade IMUs might be for more sophisticated “rubber sheeting” techniques.

Key words: strapdown inertial navigation, position, orientation, inertial measurement unit (IMU)

INTRODUCTION

Ground Penetrating Radar is used for the investigation of the subsurface or man-made structures in a variety of environments and settings. There is need for flexible and accurate antenna positioning and, to use polarization information, antenna orientation sensing. Conventional antenna positioning techniques are generally limited in their applicability. For example, wheel and string odometers do not allow for topography, “rubber sheeting” (Bochicchio, 1988) requires a smoothly varying topography and a nearly constant antenna speed for accurate positioning, laser theodolite surveying can only be used in areas with no obstacles and is slow unless a self-tracking theodolite is employed, and real-time kinematic GPS can only be used at large open outdoor sites. In addition, none of these techniques provide antenna orientation information.

Strapdown inertial navigation is potentially able to provide antenna position as well as orientation in real-time along arbitrary three-dimensional acquisition lines and allows fast data acquisition. Since strapdown inertial navigation is completely self-contained, i.e. it does not rely on other devices such as beacons or satellites, it can be used in virtually any environment, including indoors.

THEORY

The idea on which inertial navigation is based is essentially that of dead reckoning, where continuous information of speed and direction of motion of an object is used to update its position. To keep track of the object’s motion, strapdown inertial navigation systems use a cluster of three accelerometers and three gyroscopes attached rigidly to the object. The accelerometer-gyro cluster is commonly referred to as inertial measurement unit (IMU). The accelerometers sense the object’s acceleration when integrated once yields velocity, and integrating velocity yields distance traveled. The gyroscopes measure angular velocity thereby providing the necessary information for knowing where the accelerations are directed.

Notation

This section gives a brief overview of the notational features specific to the discussion of the theory. In the
following, F, F₁, and F₂ are arbitrary coordinate frames, \( \mathbf{v} \) is an arbitrary vector and \( (\cdot) \) is an arbitrary quantity.

\[
C^{F_1}_{F_2} = \text{coordinate frame transformation matrix relating}
F_2 \text{ to frame } F_1 \\
Q_{F_1F_2} = \text{attitude quaternion describing the orientation of}
F_2 \text{ relative to frame } F_1 \\
\mathbf{v}^F = \text{column matrix with elements equal to the}
\text{projection of } \mathbf{v} \text{ on the axes of frame } F \\
\mathbf{\bar{v}}^F = \text{skew-symmetric form of } \mathbf{v}^F, \text{ defined as the matrix}
\begin{bmatrix}
0 & -v_y & v_x \\
v_y & 0 & -v_z \\
-v_x & v_z & 0
\end{bmatrix}
\]

where \( v_x, v_y, v_z \) are the elements of \( \mathbf{v}^F \)
\[
\mathbf{v}_Q = \text{equivalent vector quaternion of } \mathbf{v}^F, \text{ having a}
\text{vector part equal to } \mathbf{v}^F \text{ and a zero scalar part}
\[
\omega_{F_1F_2} = \text{angular velocity of frame } F_2 \text{ relative to frame } F_1
\]

\[
(\cdot) = \text{first time derivative of quantity}
\]

\[
(\cdot)^2 = \text{second time derivative of quantity}
\]

### Coordinate Frames

From a theoretical point of view, inertial space serves as the
reference for the measurements of accelerometers and
gyrosopes. For practical purposes, inertial space may be
defined through a coordinate frame having its origin at the
earth’s center of mass and that is non-rotating relative to
the fixed stars (Britting, 1971). The orientation of its coordinate
axes is arbitrary. Particular orientations are discussed in
Chatfield (1997) and Roth (1999). In equations, the earth-
centered inertial frame is designated by the letter \( i \).

The earth frame, designated by the letter \( e \), is a coordinate
frame whose origin is at the earth’s center of mass and has
axes fixed to the earth. It is typically defined with one axis
parallel to the earth’s rotational axis and with the other axes
lying in the equatorial plane (Savage, 1998). The earth
frame rotates relative to the earth-centered inertial frame at
the earth’s rotation rate:

\[
\omega_{\text{w}} = 7.292115 \times 10^{-5} \text{ rad/s} \quad (\text{Chatfield, 1997})
\]

A local navigation frame needs to be defined which is used
as a reference for the position and orientation of the antenna
and describes the space associated with the GPR survey
area in a way that is suitable for the interpretation of radar
data. These objectives are met by the local level frame
(Roth, 1999) designated by the letter \( l \) (fig. 1). Its origin is
located at some suitable reference point, e.g. the starting
point of an acquisition line. Its \( z \) axis points in the direction
of the local gravity vector \( \mathbf{g} \), its \( x \) axis runs parallel to the
projection of a user chosen reference line onto the
horizontal plane, and the \( y \) axis completes the set. The
reference line can be any line connecting two points in the
survey area.

The ideal IMU would have accelerometer and gyroscope
input axes that constitute a perfectly orthogonal coordinate
frame and accelerometers that all measure at the same point.
Of course during manufacture perfect physical alignment
can never be achieved and accelerometers have finite
dimensions. The ideal IMU is reflected in the cube frame
(Roth, 1999) designated by the letter \( c \) (fig. 1). Its origin lies
at the center of the accelerometer cluster. Its axes can be
defined to be the orthogonalized accelerometer input axes.

Preferably, if the IMU was calibrated, the cube frame axes
are specified by the axes of the calibration table, which is
generally a two-axis rotation table (Stave, 1996).

The non-orthogonality (axis misalignment) of the inertial
sensors (accelerometers and gyroscopes) is accounted for by
the definition of the accelerometer and gyroscope frames
(Chatfield, 1997; Roth 1999). Their origins coincide with
that of the cube frame and their axes are specified by the
input axes of the corresponding inertial sensors.

### Attitude Parameters

The orientation of the antenna is identified with that of the
cube frame relative to the local level frame. This orientation
may be described by the yaw, pitch and roll angles, the
attitude quaternion \( Q_x \), or directly by the direction cosine
matrix \( C^l_e \), which transforms vectors from their cube frame
representation to their local level frame representation.

The yaw (\( \psi \)), pitch (\( \theta \)) and roll (\( \phi \)) angles describe three
consecutive rotations, whose order is important and that
would rotate the local level frame into the cube frame
(Enders, 1959) as depicted in figure 1. The yaw angle
describes the azimuthal orientation of the antenna and the
pitch and roll angles describe its tilt.

The attitude quaternion essentially describes a single
rotation that would rotate the local level frame into the cube
frame. Given that the latter is rotated with respect to the first
through angle \( \beta \) about the unit vector \( \mathbf{u} \) (fig.1), the attitude
quaternion is of the form

\[
Q_x = \begin{bmatrix}
\cos(\beta/2) \\
\sin(\beta/2) u_1 \\
\sin(\beta/2) u_2 \\
\sin(\beta/2) u_3
\end{bmatrix} \quad (\text{Roth, 1999})
\]

The relationships between the yaw, pitch and roll angles,
the attitude quaternion and the direction cosine matrix are
detailed in Kuipers (1999).
Even though the yaw, pitch and roll angles are more intuitive than either the attitude quaternion or the direction cosine matrix, it is not advisable to use them as the primary parameters for the attitude updating (Roth, 1999). They are not defined for vertical antenna orientations (Θ = ±90 deg). In the differential equations describing their rates of change, vertical orientations are associated with singularities, which introduce instability in the updating when approaching the vertical state. The attitude quaternion and the direction cosine matrix behave well for arbitrary orientations. Consequently attitude quaternion and direction cosine matrix updating would also work when inspecting sewer walls, dams, etc. The attitude quaternion and the direction cosine matrix change according to the following differential equations (Roth, 1999; Savage 1998):

\[
\dot{\mathbf{C}}_e^f = \mathbf{C}_e^f \tilde{\omega}_e^c = \mathbf{C}_e^f \tilde{\omega}_w^e - \tilde{\omega}_wp^c \mathbf{C}_e^f \tag{3}
\]

and

\[
\dot{Q}_c = 0.5 Q_{ic} \otimes \omega_{ic}^c = 0.5 (Q_{ic} \otimes \omega_{ic}^{eq} - \omega_{ic}^f \otimes Q_{ic}) \tag{4}
\]

where \(\otimes\) denotes quaternion multiplication. Both direction cosine matrix and attitude quaternion updating are widely used in practice with virtually identical results (Savage, 1998).

Despite the shortcomings of the yaw, pitch and roll angles, they are the variables of choice for the initial alignment of the navigation system (discussed below), since they allow the azimuthal orientation of the antenna and its tilt to be considered separately.

**Measured Quantities**

When corrected for axis misalignments and biases, the IMU accelerometer output is a measure of the specific force acting along the cube frame axes, denoted by \(f'\). The specific force is defined as the difference between the inertial and gravitational acceleration (Britting, 1971). Roth (1999) showed that the specific force acting along the cube frame axes can be written as

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**Figure 1: Antenna system orientation and position (symbols are explained in the text)**
\[ f^c = C^c_i \left( \dot{s}^i - g^i + 2\omega^c_i \times s^i \right) \]  

where \( \dot{s}^i \) and \( s^i \) are the first and second time derivative of \( s^i \), which is the local level frame representation of the vector going from origin of the local level frame to that of the cube frame (fig. 1).

The corrected IMU gyroscope output is a measure of the inertially referenced angular velocity of the cube frame in cube frame coordinates \( \omega_{i}^c \). This angular velocity may be written as the sum of the cube frame representation of the earth’s angular velocity \( \omega_e \) and that of the angular velocity of the cube frame relative to the local level frame \( \omega_{lc}^c \):

\[ \omega_{i}^c = \omega_{i}^e + \omega_{lc}^c \]  

(Roth, 1999).

The resolution of commercially available IMUs varies substantially. Therefore with regard to the theory, it is useful to distinguish between high-grade IMUs which can resolve the Coriolis acceleration, i.e. \( 2\omega^c_i \times s^i \), for an antenna speed of motion as low as \( |\dot{s}| \approx 2 \text{ km/h} \) and the earth’s angular velocity \( \omega_e \), and low-grade IMUs which cannot. Accordingly, when working with a low-grade IMU the measured specific force \( f^c \) can be approximated as

\[ f^c \approx C^c_i \left( \dot{s}^i - g^i \right) \]  

and the measured angular velocity \( \omega_{i}^c \) as

\[ \omega_{i}^c \approx \omega_{i}^e. \]

The IMU that we have tested is the DMU-VG (Crossbow Technology, 1997). Having an accelerometer resolution of 0.012 m/s² and a gyroscope resolution of 0.03 deg/s, it can be classified as a low-grade IMU. A high-grade IMU is for example the TGAC-RQ (iMAR) with an accelerometer resolution as low as \( 10^{-5} \) m/s² and a gyroscope resolution of 0.0003 deg/s. This level of resolution of course also manifests itself in the TGAC-RQ’s price, which lies in the US$ 55,000 range in contrast to US$ 4,000 for the DMU-VG.

**Initial Alignment**

Before the start of motion, the initial orientation of the antenna needs to be determined. This is achieved by the so-called initial alignment.

The initial alignment of strapdown inertial navigation systems used in aircraft and missiles is usually fully analytical, meaning that the full initial orientation (tilt and azimuth) is determined computationally from the IMU output. A fully analytical initial alignment requires knowledge of the directions of the gravity vector \( g \) and the earth’s angular velocity \( \omega_e \) in the navigation frame, in this case the local level frame, and measurement of their cube frame components (Britting, 1971; Chatfield, 1997). Therefore a fully analytical initial alignment is on principle only possible with a high-grade IMU. However with regard to GPR antenna navigation, even with a high-grade IMU a problem arises from the fact that the direction of \( \omega_e \) in the local level frame is not known a priori. This problem does not exist for aircraft and missile navigation since the navigation frames used in these applications are well defined with respect to a geodetic reference earth model (Chatfield, 1997). These navigation frames however have the disadvantage that they are not well defined with respect to the GPR survey area.

These problems can be overcome by an initial alignment procedure which is partly physical and partly analytical (Roth, 1999). First, the antenna is rotated until it is aligned with the x axis of the local level frame (denoted as \( x_i \), in fig. 1). As a result of this rotation the yaw angle, describing the azimuthal antenna orientation, becomes zero. Second, the tilt of the antenna system, as described by the pitch and roll angles, is determined analytically from the IMU accelerometer output using the knowledge that the gravity vector \( g \) runs parallel to the z axis of the local level frame (denoted as \( z_i \) in fig. 1). From the accelerometer output an estimate of the gravity magnitude can be calculated as well which is used to correct the accelerometer output for gravity once the antenna is in motion.

It should be noted that when working with a high-grade IMU, the orientation obtained by the above procedure can be used to calculate an estimate of the earth’s angular velocity in local level frame coordinates \( \omega_{lc}^c \) from the IMU gyroscope output. Therefore subsequent fully analytical initial alignments become possible. The so-obtained estimate of \( \omega_{lc}^c \) is also used in the attitude and the velocity updating as explained below.

**Attitude, Velocity and Position Updating**

Once the antenna is in motion, the direction cosine matrix \( C^i_i \) and the attitude quaternion \( Q_b \) change according to equations (3) and (4). An updating algorithm for either the direction cosine matrix or the attitude quaternion can be constructed by solving these differential equations over the update time interval, the boundary condition being given by the respective attitude parameter at the beginning of the interval. The basic approach to this problem is to assume an analytical form for the input angular velocity in the update interval, e.g. a polynomial in time, and then use either Runge-Kutta techniques or a Taylor series expansion up to
some order to find an estimate of the respective attitude parameter at the end of the update interval. Given a low-grade IMU, the input angular velocity is the measured angular velocity of the cube frame relative to the local level frame $\omega_{u}^{l}$. As for high-grade IMUs, the input angular velocity is the measured angular velocity of the cube frame relative to the inertial frame $\omega_{w}^{l}$. It should be noted that the earth’s angular velocity in local level frame coordinates $\omega_{u}^{l}$, which enters the high-grade updating as well, is constant.

Another common approach in the design of attitude updating algorithms is to use the rotation vector concept (Roth, 1999; Savage, 1998).

Equations (5) and (7) define the primary operations performed to update the velocity $\dot{s}$. First, the measured specific force $f_{c}$ is transformed to the local level frame using the orientation information from the attitude updating. Second, the transformed specific force $f_{l}$ is corrected for gravity and, when working with a high-accuracy IMU, for the Coriolis acceleration to obtain an estimate of the acceleration $\dot{s}$. This acceleration estimate is then integrated over the update time interval to obtain a new value for the velocity $\dot{s}$. The gravity correction makes use of the gravity estimate obtained as part of the initial alignment. Due to the “short distance” nature of the antenna navigation problem, the gravity can be assumed constant over the GPR survey area. The Coriolis acceleration can be calculated from the earth’s angular velocity $\omega_{w}^{l}$ obtained from the initial alignment and an approximation for the velocity $\dot{s}$ based on extrapolation from past values. The initial value for the velocity $\dot{s}$ to be used in the updating is zero.

The antenna position may be defined by means of a vector $a$, going from the origin of the local level frame to the measuring point of the antenna. As shown in figure 1, the vector $a$ can be expressed as the sum of the vector $s$ and another vector $d$ going from the origin of the cube frame to the measuring point of the antenna, i.e.

$$a^{l} = s^{l} + C_{d}^{l}d^{c}. \tag{9}$$

Thus, the antenna position $a^{l}$ can be calculated given $s^{l}$, $d^{l}$ and knowledge of the antenna orientation, as expressed by the direction cosine matrix $C_{d}^{l}$. The components of the vector $d^{l}$ must be measured after attaching the IMU to the antenna. The position $s^{l}$ is obviously obtained from integrating the velocity $\dot{s}$. The initial value for the position $s^{l}$ to be used for this integration can be determined from the initial antenna position $a_{0}$ (fig. 1), which must be known a priori, $d^{l}$ and the initial antenna orientation.

**NAVIGATION EXPERIMENTS**

The testing of the DMU-VG consisted of two parts. First, the ability to determine antenna orientation was investigated. The second part of the testing aimed at assessing the positioning accuracy that can be obtained.

For the purpose of the experiments, a navigation software package was developed, which integrates data acquisition, processing and display. The software neglects axis misalignments and accelerometer biases since these had not been specified by the manufacturer of the DMU-VG. The gyroscope biases are estimated as part of the initial alignment. For the attitude updating, the software uses an attitude quaternion algorithm.

To evaluate the ability to determine orientation, different rotational motions were applied to the DMU-VG by hand, which ended in the same orientation as at the start of motion. The estimated final orientations were compared to the initial orientations obtained from the initial alignment. The results demonstrate that over periods of 60 seconds the accumulation of orientation error is generally below 1 degree.

For the position estimation testing, the DMU-VG was moved by hand along different known straight and curvilinear trajectories on a horizontal tabletop (fig. 2). Position errors of up to 1.35 meters (240% over the distance traveled) after only 7.73 seconds of motion were observed. This shows that the DMU-VG is far from being accurate enough to be used for GPR antenna positioning without the aid of external position or velocity information.

![Figure 2: Experimental setup for the position estimation testing](image-url)
DISCUSSION

Strapdown inertial navigation in principal offers many advantages over conventional GPR antenna positioning techniques by its ability to provide real-time three-dimensional position and orientation information. In practice however, the error accumulation characteristic of inertial navigation systems can be a problem, especially when working with a low-grade IMU, as shown by the results of the navigation experiments. The accumulation of orientation and position errors can be attributed to the presence of scale factor errors, axis misalignments, inertial sensor biases as well as inertial sensor drift (bias drift). Position errors accumulate much faster than orientation errors as a result of the double integration.

For accurate positioning, the precise calibration for scale factors, biases and axis misalignment models is indispensable (Stave, 1996). In addition, it is important to know the effect of the inertial sensor drifts alone, since they account for most of the error accumulation of a fully calibrated IMU. A way to measure this effect is to simply leave the IMU at rest after the initial alignment and calculate positions as if it was moving (Roth, 1999). In this case position errors are solely induced by inertial sensor drift. Doing so for the DMU-VG produced position errors of the order of a couple of decimeters after only 8.5 seconds. Hence calibrating the DMU-VG would still not make it suitable for self-contained positioning. This is of course not the case for high-grade IMUs, which use inertial sensors with low drift properties such as ring laser gyroscopes (Lawrence, 1998). In fact, high-grade IMUs are generally precision calibrated by their manufacturers and the gyro and accelerometer data are corrected for systematic errors before they are output.

In spite of the unsatisfactory performance of the DMU-VG with respect to self-contained positioning, the achieved level of orientation accuracy should be sufficient enough to integrate the DMU-VG with an odometer wheel. The wheel provides information about distance traveled from which position can be estimated using the orientation information of the DMU-VG. This configuration would extend the applicability of the odometer wheel to environments with topography or where changing of the profile direction due to obstacles is necessary. Frequent stops in which, apart from keeping the estimated yaw angle, the navigation is restarted could be used to keep the accumulation of orientation errors at a minimum.

Another possible use of low-grade IMUs might be for more sophisticated “rubber sheeting” techniques which make use of the velocity information. The error growth in velocity is considerably less than in position since only a single integration is required for velocity estimation.

REFERENCES


iMAR, Product Information, internet site http://www.imar-navigation.de


