Host Medium Transformation of the Early-Time Radar Response of a Buried Dielectric Target

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***Abstract*— This paper addresses the problem of predicting the early-time radar response of a low-metal content landmine buried in a lossy soil given its response in a lossless soil. To make the problem tractable, we consider plane wave scattering from a homogeneous dielectric target embedded in a layered host medium with a global loss model. Using similarity analysis and the Born approximation, a transformation law for the scattered field is derived which relates the early-time target response in the lossy host medium to that in a corresponding lossless host medium. We also present a transformation law for the target impulse response, which follows directly from the transformation for the scattered field. The derived transformation law is tested using early-time responses of a minelike target obtained from 3D finite-difference time-domain (FDTD) simulations. Based on these tests, the ability of the transformation law to predict changes in the early-time response of a non-metallic landmine as a result of changes in the electromagnetic properties of the host medium is discussed.**

Keywords-buried landmines, early-time radar response, impulse response, host medium transformation, losses, Born approximation.

I. Introduction

The motivation for the work described in this paper has been the following question: Is it possible to predict the early-time radar response of a buried low-metal content landmine given its response in another soil? In particular, we are interested in understanding the influence of losses in the soil on the early-time response. A similar problem, though for the late-time response, has been addressed by Baum [1], who derived an expression relating the free space natural frequencies of a perfectly conducting target to those in a simple lossy medium characterized by a static conductivity and a relative dielectric permittivity. Baum's transformation is an example of a well-studied procedure for EM field transformation based on similarity analysis in the Laplace domain. Similarity analysis has been used to derive transformations for tensorial Green's functions [2], [3] and incident fields [4]. In this paper, we show that a transformation law for the field scattered by a buried dielectric target can be derived in an analogous manner by relating the equivalent scattering currents using the Born approximation. To evaluate the accuracy of the derived transformation law, we simulated the response of a minelike target for both a lossless and a lossy host medium with a 3D finite-difference time-domain (FDTD) modeling program [5]. The applicability of the derived transformation law will be discussed at the end of the paper.

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II. Theory

A. Formulation of the Scattering Problem

The transformation law is derived for plane wave scattering from a homogeneous dielectric target with permittivity ε_t fully embedded in the l^{th} layer of a *n*-layered host medium. The dielectric permittivities of the layers are ε_i ($i = 1,\ldots,n$). A global loss model is introduced by considering layer conductivities that are related to the layer permittivities by $\sigma_i(\alpha) = \alpha\varepsilon_i$; the parameter α is an arbitrary positive constant with units of reciprocal time. For simplicity, the magnetic permeability of all layers is assumed to be that of vacuum, i.e. μ_0.

In the Laplace domain, the scattered electric field $\mathbf{E}^s$ at any receiver point $\boldsymbol{x}_r$ satisfies the volume integral equation

$$\mathbf{E}^s(\boldsymbol{x}_r, s; \alpha) = \iiint_{\text{target}} \tilde{\mathbf{G}}(\boldsymbol{x}_r, \boldsymbol{x}, s; \alpha)\mathbf{J}^s(\boldsymbol{x}, s; \alpha)\,dx\,dy\,dz\,, \qquad (1)$$

where s denotes the Laplace transform parameter, $\tilde{\mathbf{G}}$ is the electric tensor Green's function and $\mathbf{J}^s$ is the equivalent electric scattering current within the target. The objective of the transformation law is to express $\mathbf{E}^s(\boldsymbol{x}_r, s; \alpha)$ in terms of $\mathbf{E}^s(\boldsymbol{x}_r, s; 0)$.

B. Derivation of the Transformation Law: Lossless to Lossy Host Medium

The starting point for the derivation is the following relationship for the tensor Green's function

$$\tilde{\mathbf{G}}(\boldsymbol{x}_r, \boldsymbol{x}, s; \alpha) = \frac{s}{(s^2 + \alpha s)^{1/2}}\tilde{\mathbf{G}}\left(\boldsymbol{x}_r, \boldsymbol{x}, (s^2 + \alpha s)^{1/2}; 0\right). \quad (2)$$

This relationship follows directly from the similarity of the lossless and lossy electromagnetic field equations in the Laplace domain. This similarity is found using the same procedure as in [2] for the transformation between electromagnetic diffusion and wave propagation.

The next step is to find a similar transformation for the scattering current $\mathbf{J}^s$. This can be achieved by making use of the Born approximation, which assumes a weakly scattering target. According to this approximation, the scattering current is related to the incident field $\mathbf{E}^{\mathrm{i}}$ by

$$\begin{aligned}\mathbf{J}^{\mathrm{s}}(\boldsymbol{x},s;\alpha) &= (s\delta\varepsilon+\delta\sigma(\alpha))\,\mathbf{E}^{\mathrm{i}}(\boldsymbol{x},s;\alpha) \\ &= (s\delta\varepsilon+\delta\sigma(\alpha))\mathrm{E}^{\mathrm{i}}(\boldsymbol{x}_t,s;\alpha)\boldsymbol{p}_{\mathrm{i}}e^{-\boldsymbol{i}k_l(s;\alpha)\boldsymbol{a}_{\mathrm{i}}\boldsymbol{x}}\end{aligned} \tag{3}$$

[6], where $\delta\varepsilon=\varepsilon_t-\varepsilon_l$ and $\delta\sigma(\alpha)=-\sigma_l(\alpha)$ are the target permittivity and conductivity contrasts, $\boldsymbol{x}_t$ denotes the target location, $\boldsymbol{p}_{\mathrm{i}}$ and $\boldsymbol{a}_{\mathrm{i}}$ are the unit vectors describing the polarization and the direction of propagation of the incident plane wave, and $k_l(s;\alpha)=-\boldsymbol{i}(\mu_0\varepsilon_l)^{1/2}(s^2+\alpha s)^{1/2}$ is the wavenumber of layer *l*. Note that without loss of generality it has been assumed that the target is located at the origin of the coordinate system. Assuming further that $\boldsymbol{p}_{\mathrm{i}}$ and $\boldsymbol{a}_{\mathrm{i}}$ are independent of α, the transformation for the scattering current is found to be

$$\mathbf{J}^{\mathrm{s}}(\boldsymbol{x},s;\alpha)= \tag{4}$$
$$\frac{s\delta\varepsilon+\delta\sigma(\alpha)}{(s^2+\alpha s)^{1/2}\delta\varepsilon}\frac{\mathrm{E}^{\mathrm{i}}(\boldsymbol{x}_t,s;\alpha)}{\mathrm{E}^{\mathrm{i}}(\boldsymbol{x}_t,(s^2+\alpha s)^{1/2};0)}\mathbf{J}^{\mathrm{s}}(\boldsymbol{x},(s^2+\alpha s)^{1/2};0).$$

Substituting eqs. 2 and 4 into eq. 1 yields the wanted expression for the scattered field transformation:

$$\mathbf{E}^{\mathrm{s}}(\boldsymbol{x}_r,s;\alpha)= \tag{5}$$
$$\frac{s^2+s\left(\frac{\delta\sigma(\alpha)}{\delta\varepsilon}\right)}{s^2+\alpha s}\frac{\mathrm{E}^{\mathrm{i}}(\boldsymbol{x}_t,s;\alpha)}{\mathrm{E}^{\mathrm{i}}(\boldsymbol{x}_t,(s^2+\alpha s)^{1/2};0)}\mathbf{E}^{\mathrm{s}}(\boldsymbol{x}_r,(s^2+\alpha s)^{1/2};0).$$

An interesting way to look at this result is to introduce the vectorial target transfer function

$$\mathbf{H}(\boldsymbol{x}_r,s;\alpha)=\frac{\mathbf{E}^{\mathrm{s}}(\boldsymbol{x}_r,s;\alpha)}{\mathrm{E}^{\mathrm{i}}(\boldsymbol{x}_t,s;\alpha)} \tag{6}$$

with which eq. 5 can be rewritten as

$$\mathbf{H}(\boldsymbol{x}_r,s;\alpha)=\frac{s^2+s\left(\frac{\delta\sigma(\alpha)}{\delta\varepsilon}\right)}{s^2+\alpha s}\mathbf{H}(\boldsymbol{x}_r,(s^2+\alpha s)^{1/2};0). \tag{7}$$

The time domain counterpart of eq. 7 can be found by application of the Schouten-Van der Pol theorem in the theory of the Laplace transformation [7], yielding

$$\mathbf{h}(\boldsymbol{x}_r,t;\alpha)=\left(\partial_t^2+\left(\frac{\delta\sigma(\alpha)}{\delta\varepsilon}\right)\partial_t\right)\int_{\tau=0}^{\infty}U_{-1}(t,\tau;\alpha)\mathbf{h}(\boldsymbol{x}_r,\tau;0)d\tau\,, \tag{8}$$

where the kernel function U_{-1} is given by

$$U_{-1}(t,\tau;\alpha)=-\int_0^{\tau}U_0(t,\tau';\alpha)d\tau' \tag{9}$$

with

$$U_0(t,\tau;\alpha)=e^{-0.5\alpha t}I_0\left(0.5\alpha(t^2-\tau^2)^{1/2}\right)S(t-\tau)\,. \tag{10}$$

Here I_0 denotes the modified Bessel function of the first kind and order zero and *S* is the Heaviside unit step function. Eq. 8 relates the target impulse response $\mathbf{h}(\boldsymbol{x}_r,t;0)$ for the lossless host medium to the target impulse response $\mathbf{h}(\boldsymbol{x}_r,t;\alpha)$ for the corresponding lossy host medium.

III. Comparative FDTD Simulations

To illustrate and verify the presented theory, we simulated the axial response of a minelike target embedded in an unbounded homogeneous medium. The target is a circular dielectric disk with a diameter of 10 cm, a height of 4 cm and has a relative permittivity of 5. The host medium relative permittivity was set to 6.25 and for the lossy case a conductivity of 20 mS/m was considered. This corresponds to an α of 0.36 ns^{-1}. For the incident plane wave we chose a linear polarization in the *x*-direction and a Ricker wavelet time function (2^{nd} derivative of a Gaussian pulse) with a peak amplitude frequency of 650 MHz. Accordingly, all simulated responses shown in this paper refer to the *x*-component of the scattered field. The incident fields at the target location were obtained by simply repeating the simulations without the target.

Fig. 1 shows the simulated target response at a distance of 50 cm above the target for both the lossless and the lossy host medium. Note that the losses do not just result in a decrease in amplitude but also a change in pulse shape. In Fig. 2, we show the axial impulse response $\mathrm{h}(\boldsymbol{x}_r,t;0)$ of the target in the lossless host medium obtained by subset selection deconvolution [8] of the simulated incident field from the simulated scattered field. The impulse response consists of two discrete differential operators, the first of which corresponds to backscatter from the top of the target and the second to backscatter from the bottom of the target. Evaluation of the integral in eq. 8 for this target impulse response gives the time function shown in Fig. 3. The result indicates that the integration with the kernel function U_{-1} is effectively equivalent to a double integration followed by a multiplication with an exponential decay function, which agrees with the target impulse response model presented in [6]. Subsequent differentiation of the integration result as prescribed in eq. 8 yields the transformed impulse response $\mathrm{h}(\boldsymbol{x}_r,t;\alpha)$ shown in Fig. 4. Finally, convolving the transformed impulse response with the incident field in the lossy host medium produces the transformed target response of Fig. 5. For comparison, the simulated target response for the lossy host medium is displayed as well, showing that the transformation law accurately predicts the changes in the target response caused by the losses.

IV. Discussion

Understanding the influence of soil properties on the radar response of buried targets is of utmost importance for radar landmine detection and identification. To this end, the presented transformation law describes how the early-time response of a dielectric target embedded in a lossless host medium is related to its early-time response in a lossy host medium. The relationship is fairly simple and hence is well suited to gain insight in how the target response changes as a result of losses. Nevertheless, the applicability of the presented transformation law is limited by a number of factors. First, the global loss model introduced in section II is not applicable to a host medium consisting of an air and a ground layer. Second, low-metal content landmines have internal structure (detonator, explosive, air gaps, etc.) and as such cannot be treated as homogeneous dielectric targets without careful consideration of the implications that this might have. And third, the reliance on the Born approximation limits the range of soils and radar frequencies for which the transformation law holds. Therefore, measurements of radar landmine responses for different soils remains indispensable for a complete understanding of the problem.

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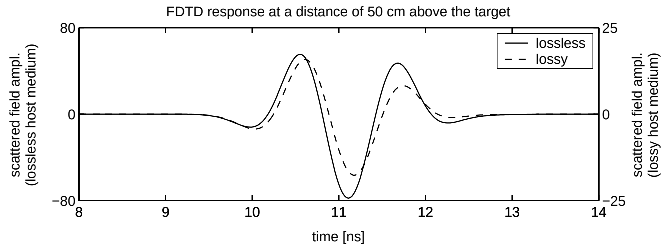

Figure 1. Simulated axial response of the example target in the lossless ($\varepsilon_r = 6.25$) and the lossy ($\varepsilon_r = 6.25, \sigma = 20$ mS/m) host medium.

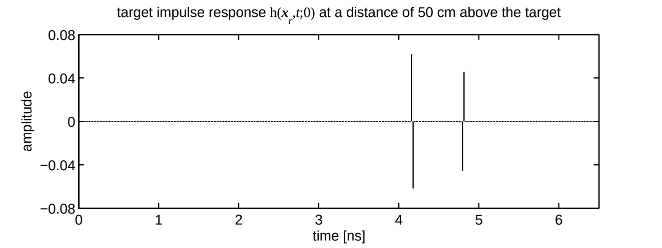

Figure 2. Axial impulse response of the example target in the lossless host medium ($\varepsilon_r = 6.25$) computed by the subset selection deconvolution.

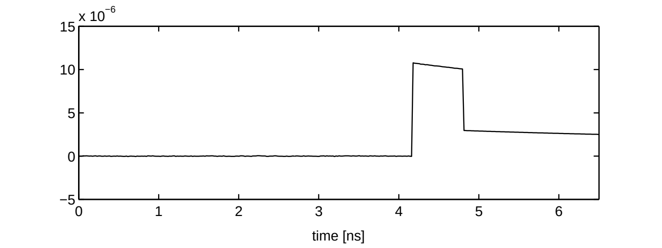

Figure 3. Evaluation of the integral in eq. 8 for the impulse response shown in Fig. 2.

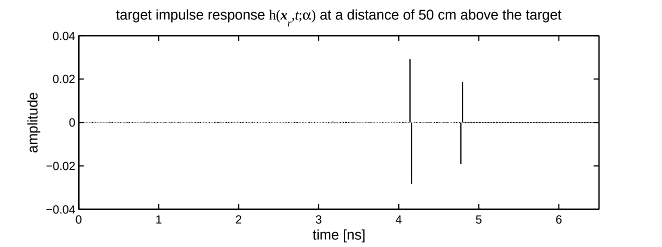

Figure 4. Axial impulse response of the example target in the lossy host medium ($\varepsilon_r = 6.25, \sigma = 20$ mS/m) as predicted by eq. 8 ($\alpha = 0.36\,\text{ns}^{-1}$).

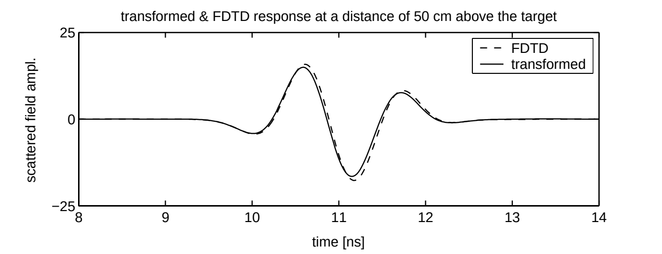

Figure 5. Transformed and simulated axial response of the example target in the lossy host medium ($\varepsilon_r = 6.25, \sigma = 20$ mS/m).